

Binary Trees

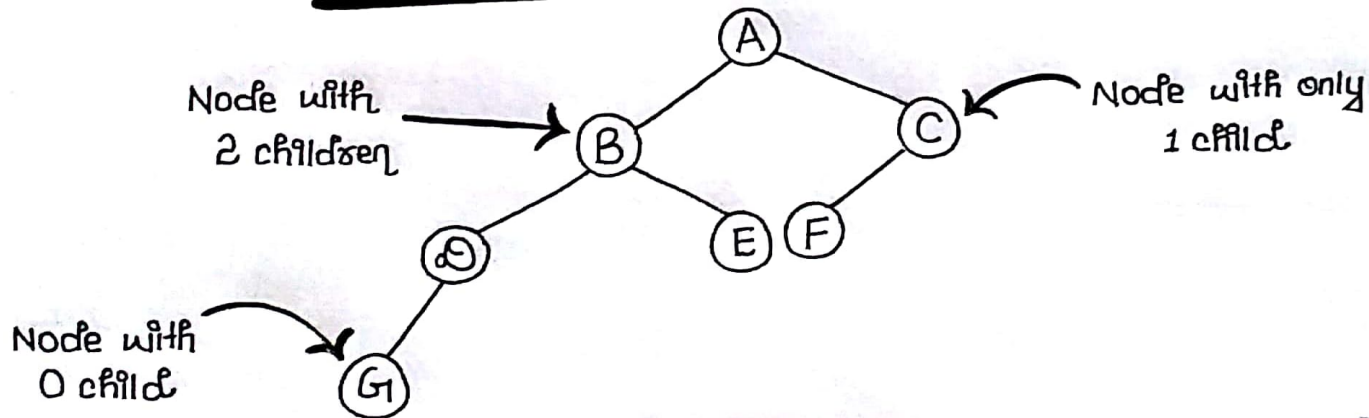


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Definition:

- Binary tree is a special type of tree data structure in which every node can have at most 2 children i.e. each node can have either 0 child or 1 child or 2 children but not more than 2 children.

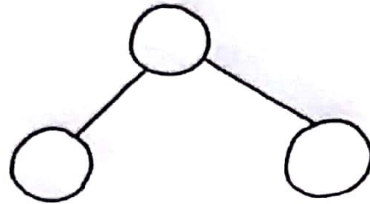
Example:



Unlabelled Binary Trees:

- A binary tree is unlabelled if its nodes are not assigned any label.

- Example:



is an unlabelled binary tree.

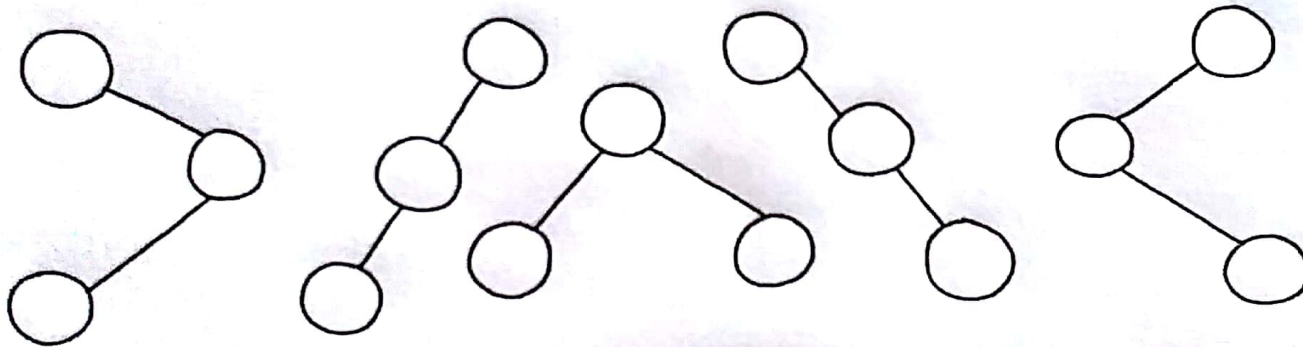
Number of different binary trees possible with 'n' unlabelled nodes

$$= \frac{2^n C_n}{n+1}$$

Illustration:

with 3 unlabelled nodes, number of different binary trees possible

$$\begin{aligned} &= \frac{2 \times 3 C_3}{3+1} \\ &= \frac{6 C_3}{4} \\ &= 5 \end{aligned}$$

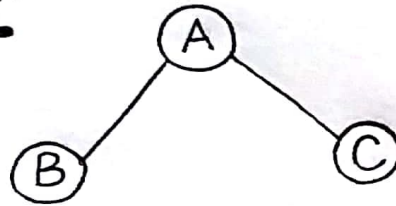


Thus, with 3 unlabelled nodes, 5 different binary trees are possible.

Labelled Binary Trees:

- A binary tree is labelled if every node is assigned a label

- Example:



is a labelled binary tree

$$\begin{aligned} \text{Number of binary trees possible} &= \frac{2^n C_n \times n!}{n+1} \\ \text{with } n \text{ labelled nodes} & \end{aligned}$$

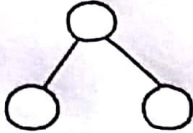
Illustration:

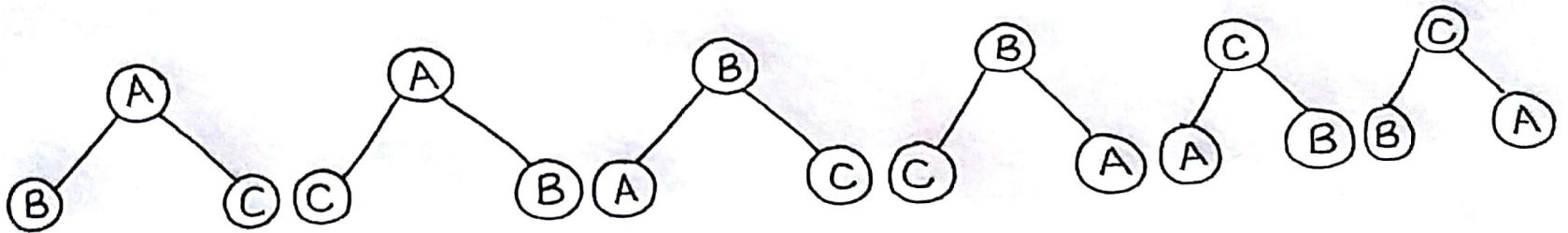
with 3 labelled nodes, number of different binary trees possible

$$= \frac{2 \times 3}{3+1} C_3 \times 3!$$

$$= \frac{6}{4} C_3 \times 3!$$

$$= 30$$

Example: The structure  will give $3! = 6$ different labelled structures.

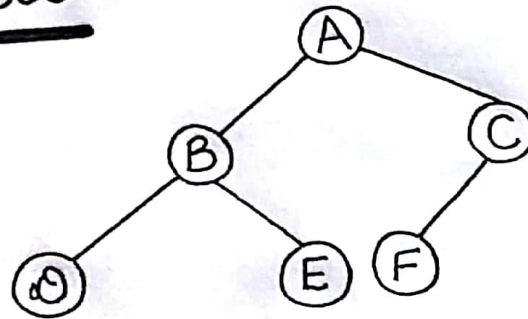


Types of Binary Trees

1. Rooted Binary Tree:

- A rooted binary tree is one that has a root node and every node has at most 2 children.

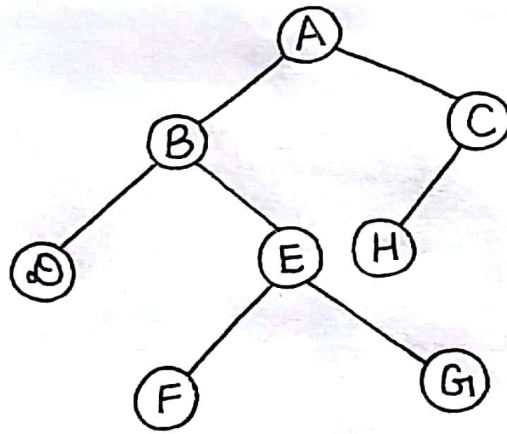
Example:



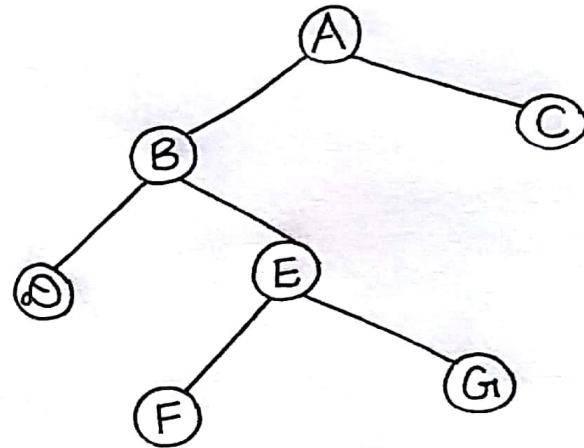
2. Full/Strictly Binary Tree:

- A full/strictly binary tree is one in which every node has either 0 or 2 children.

Example:



✗
Not Full/Strictly Binary Tree

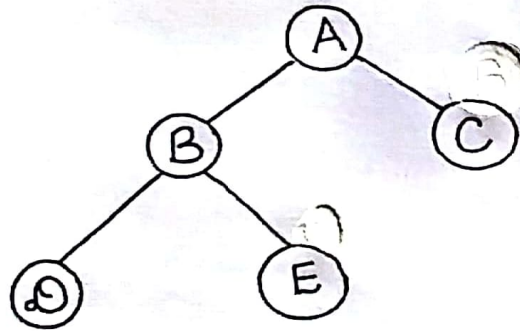


✓
Full/Strictly Binary Tree

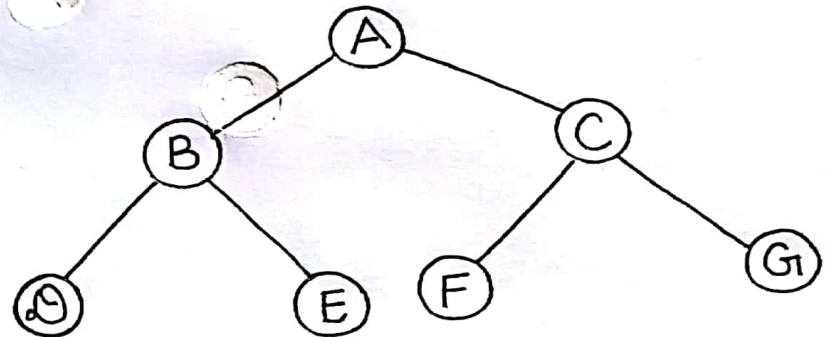
3. Complete / Perfect Binary Tree:

- A complete / perfect binary tree is one in which every internal node has exactly two children and all the leaf nodes are at same level

Example:



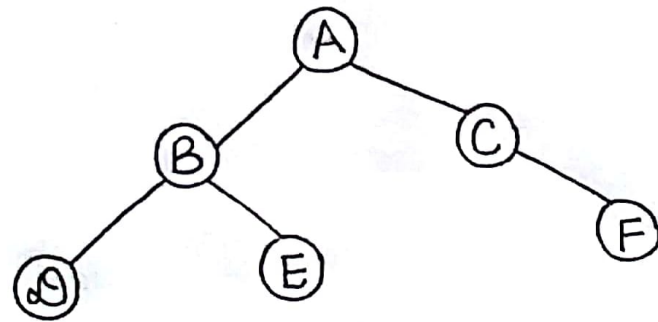
X
Not a complete / perfect
Binary Tree



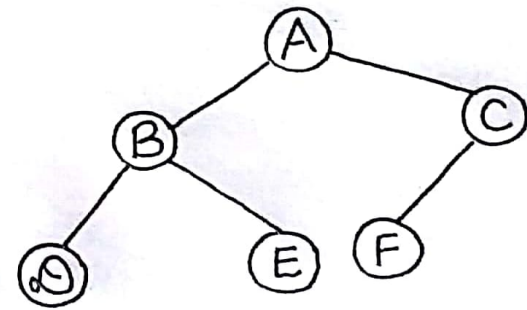
✓
Complete / Perfect Binary Tree

4. Almost Complete Binary Tree:

- An almost complete binary tree is one in which all the levels are completely filled except possibly the last level and the last level must be strictly filled from left to right.
- Example:



X
Not almost complete
Binary Tree



✓
Almost complete
Binary Tree

5. Skewed Binary Trees:

- A skewed binary tree is one where all the nodes except one node have one and only one child. The remaining node has no children.

OR

A skewed binary tree is a binary tree of n nodes such that its depth is $n-1$.

- Example:

